MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 2 - 2016

Discrete Random Variables and Applications of Differentiation

Resource Rich - SOLUTIONS

ALL SAINTS' COLLEGE

[2 marks each for each correct multiple choice answer]

1 Probability distribution =

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

Ordered pairs for the function = $\left(1, \frac{1}{36}\right), \left(2, \frac{3}{36}\right), \left(3, \frac{5}{36}\right), \left(4, \frac{7}{36}\right), \left(5, \frac{9}{36}\right), \left(6, \frac{11}{36}\right)$

$\left(5, \frac{8}{36}\right)$ is not one of the ordered pairs listed.

$\therefore$ **B**

2 $E(X) = 5 \times 0.4 + 6 \times 0.3 + 7 \times 0.2 + 8 \times 0.1$

$= 6$

$E(X^2) = 5^2 \times 0.4 + 6^2 \times 0.3 + 7^2 \times 0.2 + 8^2 \times 0.1$

$= 37$

$\text{Var}(X) = 37 - 6^2 = 1$

$\therefore$ **C**

3 Probability distribution =

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

$p(2 < x \leq 5) = p(3) + p(4) + p(5)$

$= \frac{5}{36} + \frac{7}{36} + \frac{9}{36}$

$= \frac{7}{12}$

$\therefore$ **B**

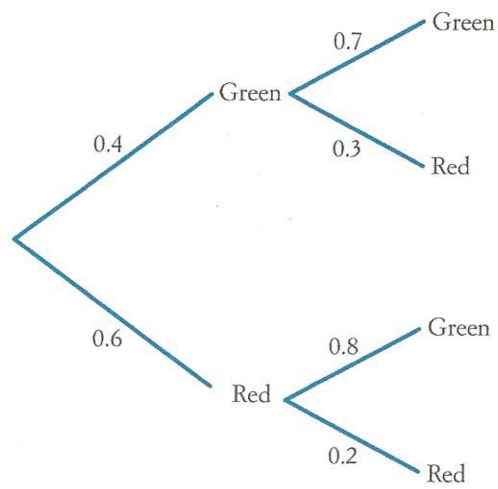
1. B
2. C
3. B
4. A
5. D
6. D
7. C
8. B
9. E
10. A

4 $E(X) = 0 \times 0.1 + 2 \times 0.15 + 4 \times 0.15 + 6 \times 0.25 + 8 \times 0.35$

$= 5.2$

∴ A

5 Tree diagram for this situation =



$P(x = 0) = 0.4 \times 0.7 = 0.28$

$P(x = 1) = 0.6 \times 0.8 + 0.4 \times 0.3 = 0.6$

$P(x = 2) = 0.6 \times 0.2 = 0.12$

$E(X) = 0 \times 0.28 + 1 \times 0.6 + 2 \times 0.12$

$= 0.84$

∴ D

6 $y = 3x^3 + 4x^2 + 5$

$y' = 9x^2 + 8x$

When $x = 2$

$y' = 52$

$\delta y = 52 \times 0.03$

$= 1.56$

∴ D

7 $y = 4x\cos(x)$

$y' = 4\cos(x) - 4x\sin(x)$

$y'' = -4\sin(x) - 4\sin(x) - 4x\cos(x)$

$= -8\sin(x) - 4x\cos(x)$

∴ C

8 $y = 2x^3 + 12x^2 - 18x - 5$

$y' = 6x^2 + 24x - 18$

$y'' = 12x + 24$

concave upwards when $y'' > 0$

$12x + 24 > 0$

$12x > -24$

$x > -2$

$\therefore$ **B**

9 $y = x^3 - 6x^2 - 36x + 9$

$y' = 3x^2 - 12x - 36$

For stationary points, $y' = 0$

$3x^2 - 12x - 36 = 0$

$x^2 - 4x - 12 = 0$

$(x - 6)(x + 2) = 0$

$x = -2, 6$

$y'' = 6x - 12$

When $x = -2$

$y'' = 6 \times -2 - 12 < 0$

Maximum at $(-2, 49)$

When $x = 6$

$y'' = 6 \times 6 - 12 > 0$

Minimum at $(6, -207)$

$\therefore$ **E**

10 Let the two numbers be x and y.

Then $xy = 72$ and the sum $S = 2x + 4y$

$$y = \frac{72}{x}$$

Substitute into $S = 2x + 4y$

$$S = 2x + 4\left(\frac{72}{x}\right)$$

$$S = 2x + \frac{288}{x}$$

$$\frac{dS}{dx} = 2 - \frac{288}{x^2}$$

Stationary point when $\frac{dS}{dx} = 0$,

$$2 - \frac{288}{x^2} = 0$$

$2x^2 = 288$

$x^2 = 144$, since x is positive

$x = 12$

$y = \frac{72}{12}$

$y = 6$

$\therefore$ **A**

11 [7 Marks]

a Construct the probability distribution.

x	2	5	7
$p(x)$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$

[2 marks]

b $E(X) = 2 \times \frac{2}{5} + 5 \times \frac{1}{5} + 7 \times \frac{2}{5}$

$= 4\frac{3}{5}$ **[1 mark]**

c $E(X + b) = E(X) + b$

Let $Y = X + 3$

$E(Y) = E(X) + 3$ **[1 mark]**

$= 4\frac{3}{5} + 3$

$= 7\frac{3}{5}$ **[1 mark]**

d $E(bX) = bE(X)$

Let $Z = 5X$

$E(Z) = 5E(X)$ **[1 mark]**

$= 4\frac{3}{5} \times 5$

$= \frac{23}{5} \times 5$

$= 23$ **[1 mark]**

12 [6 Marks]

a Probability distribution =

x	1	2	3	4	5
$p(x)$	$\frac{k}{2}$	$\frac{2k}{3}$	$\frac{3k}{4}$	$\frac{4k}{5}$	$\frac{5k}{6}$

[3 marks]

b $\Sigma p(x) = 1.$

$$\frac{k}{2} + \frac{2k}{3} + \frac{3k}{4} + \frac{4k}{5} + \frac{5k}{6} = 1$$ **[1 mark]**

$$\frac{60}{213} \times \left(\frac{k}{2} + \frac{2k}{3} + \frac{3k}{4} + \frac{4k}{5} + \frac{5k}{6}\right) = \frac{60}{213} \times 1$$ **[1 mark]**

$$\frac{30k + 40k + 45k + 48k + 50k}{60} = 1$$

$$\frac{213k}{60} = 1$$

$$k = \frac{60}{213}$$

$$k = \frac{20}{71}$$ 0.282 **[1 mark]**

13 **[4 Marks]**

The sum of the probabilities must be 1.

$0.15 + 0.25 + a + b = 1$ **[1 mark]**

$a + b = 0.6$

$a = 0.6 - b$

$E(X) = (0 \times 0.15) + (1 \times 0.25) + 2a + 3b$

$1.93 = 0.25 + 2a + 3b$

$2a + 3b = 1.68$ **[1 mark]**

$2(0.6 - b) + 3b = 1.68$

$1.2 - 2b + 3b = 1.68$

$b = 0.48$ **[1 mark]**

$a = 0.6 - 0.48 = 0.12$ **[1 mark]**

14 **[5 Marks]**

a The average age of the population i decreasing.

[1 mark]

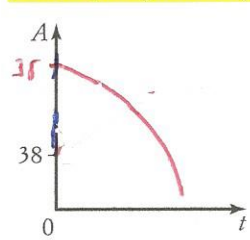
b The rate at which the age of the population is decreasing, is slowing down.

[1 mark]

c [1 mark] for concave downwards for x

[1 mark] for increasing curve

[1 mark] for y-intercept of 38



15 **[3 Marks]**

$h = 2x - 16 - 0.05x^2$

$h' = 2 - 0.1x$ **[1 mark]**

Stationary point when $h' = 0$

$2 - 0.1x = 0$

$x = 20$

$h'' = -0.1 < 0$ maximum **[1 mark]**

When $x = 20$

$h = 2 \times 20 - 16 - 0.05(20)^2$

$= 4$ m **[1 mark]**

16 **[7 Marks]**

Volume $= \pi r^2 h$

$500 = \pi r^2 h$ **[1 mark]**

$h = \dfrac{500}{\pi r^2}$ **[1 mark]**

Surface area $= 2\pi r^2 + 2\pi rh$

$A = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$ **[1 mark]**

$= 2\pi r^2 + \dfrac{1000}{r}$

$\dfrac{dA}{dr} = 4\pi r - \dfrac{1000}{r^2}$ **[1 mark]**

$= 0$ when $4\pi r - \dfrac{1000}{r^2} = 0$ **[1 mark]**

$4\pi r = \dfrac{1000}{r^2}$

$r^3 = \dfrac{1000}{4\pi}$

$r = \sqrt[3]{\dfrac{1000}{4\pi}}$

$r = 4.3$ correct to 2 sig. fig. **[1 mark]**

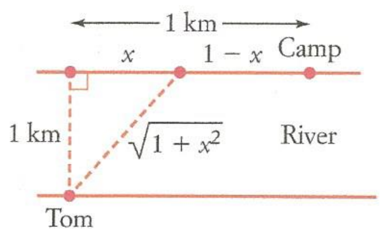
$\dfrac{d^2A}{dr^2} = 4\pi + \dfrac{1000}{r^3}$

> 0 for all $r \geq 0 \therefore$ minimum **[1 mark]**

i.e. radius of can is 4.3 cm

17 **[6 Marks]**

Swim to a point approximately 0.89 km along the river towards his camp and then walk approximately 0.11 km to his camp. This will take approximately 42 minutes 22 seconds.



Swim: 2 km/h, Walk: 3 km/h

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time = Swim time + Walk time

$$T = \frac{\sqrt{1+x^2}}{2} + \frac{1-x}{3}$$ **[1 mark]**

$$\frac{dT}{dx} = \frac{x}{2\sqrt{1+x^2}} - \frac{1}{3}$$ **[1 mark]**

$$= 0 \quad \text{when} \quad \frac{x}{2\sqrt{1+x^2}} - \frac{1}{3} = 0$$ **[1 mark]**

$$\frac{x}{2\sqrt{1+x^2}} = \frac{1}{3}$$

$$3x = 2\sqrt{1+x^2}$$

$$9x^2 = 4(1+x^2)$$

$$9x^2 = 4 + 4x^2$$

$$5x^2 = 4$$

$$x^2 = \frac{4}{5}$$ **[1 mark]**

$$x = \frac{2}{\sqrt{5}} \approx 0.89 \quad \text{since } 0 \le x \le 1$$ **[1 mark]**

Substitute into T to find $T \approx 0.706$ hours ≈ 42 minutes 22 seconds **[1 mark]**

18 **[9 Marks]**

a $s = 2t^3 + 9t - 8$

$$\frac{ds}{dt} = 6t^2 + 9$$ **[1 mark]**

When $t = 2$

$$\frac{ds}{dt} = 6(2)^2 + 9$$ **[1 mark]**

$= 33$ m/s

b 1 s

$$\frac{ds}{dt} = 6t^2 + 9$$

When $\frac{ds}{dt} = 15$

$$6t^2 + 9 = 15$$ **[1 mark]**

$6t^2 = 6$

$t^2 = 1$

$t = \pm 1$ **[1 mark]**

but $t \geq 0$, so $t = 1$ **[1 mark]**

c $\dfrac{d^2s}{dt^2} = 12t$ **[1 mark]**

When $t = 2$

$\dfrac{d^2s}{dt^2} = 12(2)$ **[1 mark]**

$= 24\ \text{m/s}^2$

d $12\ \text{m/s}^2$

velocity = 15 when $t = 1$ **[1 mark]**

$\dfrac{d^2s}{dt^2} = 12(1)$

$= 12$ **[1 mark]**

19 **[3 Marks]**

Concave upwards for $x < 0$ **[1 mark]**

Concave downwards for $x > 0$ **[1 mark]**

Decreasing curve **[1 mark]**